ABSORPTION, REFLECTION, AND DISPERSION CONSTANTS OF QUARTZ

By W. W. Coblentz

An accurate knowledge of the transmission of quartz in the infra-red is essential in order to determine spectral energy curves of a black body with a quartz prism. As mentioned elsewhere, the program of investigation [1] of the radiation constants of a black body includes the employment of a quartz prism, which has a much larger dispersion than fluorite. Unfortunately, unlike fluorite, the absorption in quartz becomes quite marked at 2μ and is practically complete beyond 3μ. However, for the spectral region from the remote ultra-violet (0.25μ) to 1.7μ in the infra-red, quartz has the remarkable property of practically perfect transparency for the thicknesses (3 cm) which are ordinarily used in optical investigations. This is illustrated in the present research, where it is shown that after eliminating the losses for reflection at the interface, quartz air, the transmission is 100 per cent, if we admit an error of 2 parts in 1000. This property of great transparency in one spectral region, and great opacity in a closely adjoining region, requires the employment of thick samples of quartz in order to determine accurately the transmission in the intervening region. The whole investigation requires a special procedure, which one would not ordinarily undertake. The data obtained in the present investigation are therefore published with the hope that they may be of use to others.

A preliminary [2] examination of several samples of quartz was recently published. One sample consisted of two plates, each 6.5 mm in thickness. Inadvertently, as published, the thickness is stated to be 13 mm, with no mention that it represented two

[1] This Bulletin, **10**, p. 2; 1913.

[2] This Bulletin, **9**, p. 81; 1912.

plates. This explains the low transmission, caused by the four reflecting surfaces, as shown in the present paper, Fig. 1, curve *a*, where these data are reproduced. A second sample used in the preliminary examination consisted of a quartz prism, the ends of which were polished, through which the light passed, thus giving a layer 52 mm in thickness. The transmission (uncorrected for surface reflection) through this prism, given in Fig. 1, curve *b*, is of the order of 91.5 per cent, in the region of 1μ, which is the value found in the present work, using other material.

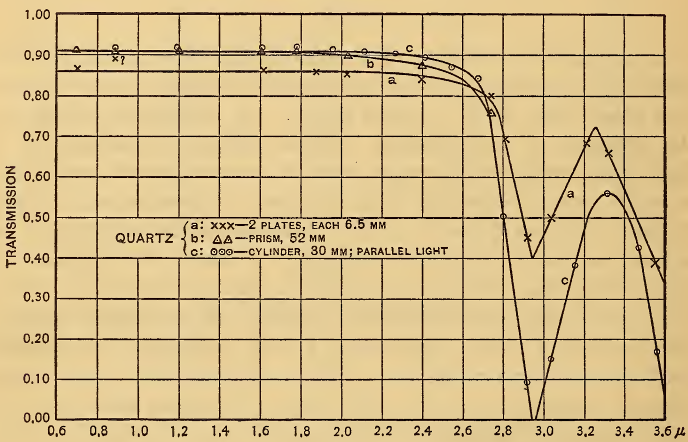

FIG. 1

In the preliminary test the source of radiation (a Nernst glower) was placed at a distance of about 10 cm from the spectrometer slit, through which the light passed, forming a narrow (8 to 10 mm wide) somewhat divergent beam upon the collimating mirror of the spectrometer. The quartz samples stood close in front of the spectrometer slit. The accuracy attained in the observations was not very high (1 per cent), owing to unsteadiness of the bolometer. In view of the fact that the light was not parallel [3]

[3] This, however, could do but little more than shorten the optical path. By actual measurement no variation in the width of the cone of light falling upon the collimating mirror could be detected when the thick quartz plate (the prism with polished ends) was placed before the slit. In the present research a mirror spectrometer fluorite prism and vacuum bolometer were used, as described in previous papers.

and that the transmission was not taken along the optic axis of the prism the investigation was undertaken anew, using two especially prepared cylinders of clear quartz. One cylinder was 50 mm in diameter and 29.925 mm in length. The faces were cut at an angle of about 40° to the optic axis and they had an unusually fine polish. The faces were plane to a fraction of a wave length of light, and they were parallel to within 1′ of arc. This sample of quartz was perfectly free from smokiness, except at the extreme edge, which was shielded from radiation.

The second cylinder of quartz was 50 mm in diameter and 27.915 mm in thickness. The faces were cut perpendicular to the axis to within 49.5′ ± 0.5′, the error in plane parallelism being about 0.5′. This sample also showed faint traces of smokiness near the edge which was covered. The faces were not quite so highly polished as the first sample, which may account for the lower transmission (greater scattering) in the visible spectrum.

The source of energy was a seasoned Nernst glower operated on a storage battery. Fortunately, the observations happened to come on several calm, cloudy days, when the vacuum bolometer and galvanometer were perfectly steady, and the glower likewise was not affected by air currents. The rays from the glower were made parallel by means of a 50-cm focal length silvered mirror, and thence passed to a second mirror, 90 cm in focal length, which brought the rays to focus upon the spectrometer slit. A blackened diaphragm having an opening of about 3 by 3.5 cm was placed in the path of parallel rays. The quartz cylinder was mounted upon a suitable stage, sliding black and forth, close to and in the rear of the diaphragm (i. e., between the diaphragm and the second mirror). The ratio of the galvanometer deflection, observed when the quartz plate was over the opening in the diaphragm, to the deflection caused by the rays passing through the diaphragm without obstruction by the quartz, gave the transmission (*Tr*, Table 1). Atmospheric disturbances were unusually small, so that the galvanometer could be read to 0.1 mm, thus producing an accuracy of several parts in 1000 instead of a few parts in 100, which is the usual record of radiometric work.

The transmission curve, *c*, is given in Fig. 1, and the observations (*Tr*) are given in columns 3 and 6 of Table 1. In the transparent

region to 1.6μ the transmission is about 91.5 per cent, as previously observed on the prism which was almost twice as thick, with the (divergent) rays passing across the optic axis. Eliminating surface reflection from the observations on the two thin plates, previously investigated, curve *a*, Fig. 1, the data are in agreement with those of the thick plates.

In Table 1, columns 4 and 7, are given the true transmissions, T_0, of these two plates of quartz. These values are obtained by applying the well-known Fresnel formula for vitreous reflection, $R=\left(\frac{n-1}{n+1}\right)^2$ where n is the refractive index. The refractive indices (Table 3) of Carvallo and of Paschen were used.

TABLE 1

Transmission of Quartz

λ in mm	Reflection correction $\frac{(1-R)^2}{(1+R^2)}$	Quartz cylinder (t=29.925 mm)			Quartz cylinder (t=27.915 mm)		
		T_r (observed)	T_0	T_0^1	T_r (observed)	T_0	T_0^1
0.0005893	0.9125	0.9060	0.9929	0.9960	0.9055	0.9923	0.9958
.0008820	.9143	.9112	.9966	.9997	.9070	.9920	.9955
.0011971	.9153	.9145	.9991	1.0022	....	....	....
.0016132	.9167	.9138	.9969	1.0000	.9135	.9966	1.0000
.0017835	.9173	.9143	.9967	.9998	.9140	.9964	.9999
.0019518	.9179	.9120	.9936	.9967	.9126	.9942	.9976
.0021128	.9185	.9092	.9898	.9929	.9106	.9914	.9948
.0022654	.9192	.9037	.9831	.9862	.9050	.9846	.9880
.0024098	.9199	.8967	.9748	.9778	.8979	.9761	.9795
.0025458	.9206	.8720	.9473	.9502	.8766	.9522	.9555
.0026120	.9209	....	....	....	.8626	.9367	.9399
.0026757	.9213	.8445	.9168	.9195	.8512	.9239	.9272
.0027392	.9216	....	....	....	.7757	.8417	.8446
.0028010	.9219	.5050	.5477	.5494	.5319	.5769	.5789
.0029213	.9227	.0917	.0994	.0997	.0748	.0811	.0814
.0030373	.9234	.153	....	....	.1426	.1544	.1549
.0031501	....	.384	....	....	.3776	....	....
.0033132	....	.560	....	....	.5558	....	....
.0034190	....	....	....	....	.5285	....	....
.0034700	....	.430	....	....	.4573	....	....
.0035690	....	.171	....	....	.1895	....	....

The radiation contributed by two reflections $R^2(1-R)^2$ within the quartz plate is negligible in most work. If there were no errors of observation and no scattering of light, the values should be

$T_0 = 1$ in the spectral region where there is no absorption. From the present observations this appears to be true to about 2 parts in 1000 for the region of transparency from 0.6 to 1.8μ. The fifth and eighth columns of Table 1 give the transmission T_0 on the assumption that there is perfect transmission up to and at $\lambda = 1.6132\mu$, i. e., all the observed values are divided by 0.9969, which is the observed value, T_0, at $\lambda = 1.6132\mu$. The deviations from $T_0 = 1$ are then found to be far smaller than the average experimental errors which usually enter into such work.

TABLE 2

Transmission of Quartz—Extinction Coefficient

λ in mm	Quartz cylinder (t=29.925 mm) κ	Quartz cylinder (t=27.915 mm) κ	κ—Mean value
0.0016132	0.000 000 009	0.000 000 010	0.000 000 009
.0017835	.000 000 010	.000 000 012	.000 000 011
.0019518	.000 000 022	.000 000 021	.000 000 022
.0021128	.000 000 038	.000 000 035	.000 000 037
.0022654	.000 000 068	.000 000 066	.000 000 067
.0024098	.000 000 108	.000 000 109	.000 000 109
.0025458	.000 000 243	.000 000 235	.000 000 239
.0026120	....	.000 000 323	.000 000 323
.0026757	.000 000 410	.000 000 400	.000 000 405
.0027392	....	.000 000 894	.000 000 894
.0028010	.000 002 98	.000 002 92	.000 002 95

In passing it is desirable to record the very marked reflection caused by fine particles of dust. One sample of quartz, which had stood uncovered for a day, transmitted over 1 per cent (depending upon the wave length) less than the usual value (89.8 instead of 91.35 per cent). Examined in front of a bright incandescent lamp, the surfaces showed faint tarnishing and dust. After thoroughly polishing the surfaces with a soft cloth the normal transmission (91.4 per cent) was again observed.

The absorption index (extinction coefficient) κ is computed from the equation $A = 1 - e^{-al}$, where $a = 4\pi \frac{n\kappa}{\lambda}$. Here the thickness, l, of the plate and the wave length, λ, are in millimeters. The data for the two samples, computed from columns 4 and 7 of

Table 1, are given in Table 2, the values being in close agreement where the true absorption of quartz becomes important. A large absorption band of quartz occurs at 2.95μ; and a small depression at 2.6μ (see Fig. 2) was observed in the transmission of both samples of quartz. The sample of quartz, 27.915 mm in thickness, was not so highly polished as the first specimen. This would cause more scattering of light, and an apparently high extinction coefficient, especially in the visible spectrum.

TABLE 3

Quartz Reflection

[In this table R is the reflection which occurs at normal incidence and R_1 and R_2 are the reflection coefficients for light polarized respectively in and perpendicular to the plane of incidence.]

λ	$(1-R)^2(1+R^2)$	$\frac{(1-R_1)^2+(1-R_2)^2}{2}$	Refractive index	λ	$(1-R)^2(1+R^2)$	$\frac{(1-R_1)^2+(1-R_2)^2}{2}$	Refractive index
0.325μ	0.9062	0.8668	1.57094	1.0417	....	0.8821	1.53442
.340	.9069	....	1.56744	1.0715	0.9149	....	1.53402
.3582	.9077	....	1.56400	1.1592	....	.8827	1.53283
.361	.9078	....	1.56348	1.2215	.9154	....	1.53201
.396	.9091	.8722	1.55815	1.3070	....	.8835	1.53090
.4046	.9094	....	1.55706	1.376	.9159	....	1.53001
.410	.9095	....	1.55649	1.4219	....	.8842	1.52942
.434	.9101	.8740	1.55396	1.528	.9164	....	1.52800
.4862	.9113	....	1.54964	1.5414	....	.8848	1.52781
.508	.9115	.8764	1.54822	1.670	.9169	....	1.52602
.5349	.9119	....	1.54663	1.6815	....	.8856	1.52583
.5893	.9125	.8780	1.54420	1.7614	....	.8860	1.52468
.6158	.9127	.8785	1.54323	1.870	.9176	....	1.52302
.643	.9129	.8789	1.54226	1.9457	....	.8872	1.52184
.6563	.9130	....	1.54182	1.999	.9181	....	1.52100
.6678	.9131	....	1.54155	2.170	.9188	....	1.51802
.686	.9133	.8794	1.54097	2.1719	....	.8886	1.51799
.7065	.9134	....	1.54048	2.3573	....	.8900	1.51449
.7435	.9136	....	1.53956	2.384	.9198	....	1.51403
.760	.9137	.8802	1.53917	2.574	.9207	....	1.51002
.7682	.9138	....	1.53890	2.6519	....	.8925	1.50824
.7711	.9138	....	1.53895	2.746	.9216	....	1.50602
.8007	.9139	.8805	1.53835	2.7993	....	.8938	1.50474
.8325	.9141	....	1.53773	2.904	.9226	....	1.50201
.8671	.9142	.8810	1.53711	3.058	.9235	....	1.49800
.9047	....	.8813	1.53649	3.0939	....	.8967	1.49703
.9335	.9145	....	1.53600				

The factors for eliminating the absorption in a wedge of quartz are determined from the equation [4]

$$\frac{I_o}{I}=\frac{\log (1-A)^{B/l}}{[(1-A)^{B/l}-1] \log \epsilon}.$$

It is derived from an integration of the equation

$$I=\frac{I_o}{h}\int_o^h \epsilon^{-a\frac{B}{h}x} dx.$$

In these equations A is the absorption $(1-T_o)$ observed in the quartz plate and l is its thickness (29.925 mm); B is the thickness of the back of the prism (i. e., the width of the face, 52 mm, which is opposite the refracting angle); and h is the vertical height of the refracting edge from the back of the prism. The observed intensity (in galvanometer deflections), is I; and I_o is the true intensity of the radiations emanating from the source.

TABLE 4

Factors for Eliminating the Effect of Absorption in a Prism Having a Base (Back Face) of 50 to 52 mm in Width

λ in mm	Factor			
	t=29.925 mm	t=27.915 mm	Mean	$\frac{\text{Mean}}{1.003}$
0.0016132	1.0028	1.0032	1.0030	1.0000
.0017835	1.0029	1.0034	1.0032	1.0002
.0019518	1.0057	1.0055	1.0056	1.0026
.0021128	1.0089	1.0082	1.0086	1.0056
.0022654	1.0150	1.0147	1.0149	1.0119
.0024098	1.0226	1.0229	1.0228	1.0197
.0025458	1.0483	1.0467	1.0475	1.0444
.0026120	....	1.0628	1.0628	1.0596
.0026757	1.078	1.076	1.077	1.074
.0027392	....	1.117	1.117	1.114
.0028010	1.619	1.605	1.612	1.607

In Fig. 2 curve A, gives the true transmission (T_o of Table 1) of the thickest plate of quartz. The absorption begins at 1.8μ. Curve B, Fig. 2, gives the factors to be used in eliminating the effect of this absorption in a prism, the back face of which is 50 to 52 mm in thickness. Numerical data for eliminating this absorption, which begins at about 1.9μ, are given in Table 4. This table shows

[4] Paschen, Sitzber. Akad. Wiss., Berlin, 22, p. 405, 1899.

that, for example, at 2.739μ the observed intensity must be increased by 11.4 per cent in order to correct for the loss by absorption in the prism. The last column in Table 4 gives the correction for absorption in the prism on the assumption that the error of the present observation is 0.3 per cent. It is obtained by dividing all the mean values by 1.003.

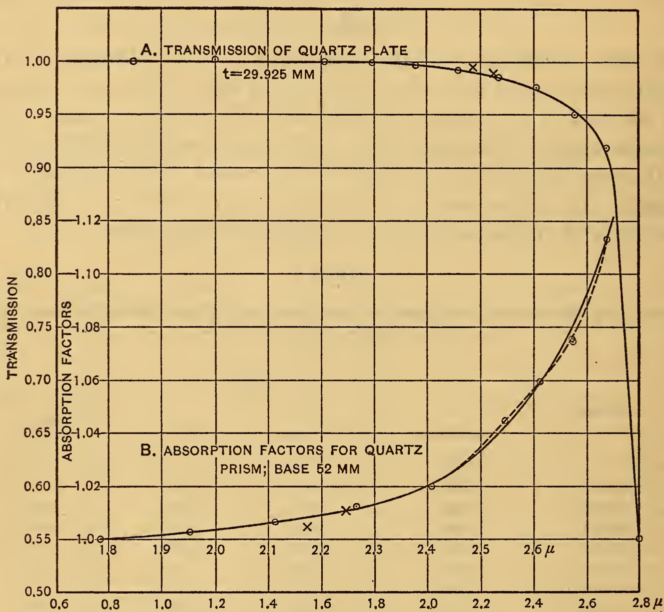

FIG. 2

A further increase in the value of I is, of course, necessary for the loss by reflection from the prism faces with variation in the angle of incidence which is required in order to cause the rays to pass through the prism at minimum deviation. The values for reflection were computed by means of the Fresnel formulas,

$$R_1 = \frac{\sin^2(i-r)}{\sin^2(i+r)} \quad R_2 = \frac{\tan^2(i-r)}{\tan^2(i+r)},$$

which give the variation in reflection with variation in the angle of incidence. The angles of incidence used in this computation are those required in order that the rays will pass at minimum diviation through a quartz prism having an angle of 59° 51′ 21″. These values may be used without serious errors with a prism having a refracting angle which differs by 5′ to 6′ from the one used in the present computations. The numerical data for eliminating the variation in reflection with variation in angle of incidence are given in column 3 of Table 3. For a prism having a refracting angle of 60° the values should be decreased by 0.0005.

If the spectral energy measurements are made with a radiometer which is covered with a quartz window then a still further correction for reflection (also for absorption) must be made to the observations. This correction for reflection is the same as the one applied in determining the transmission of quartz. The numerical values for correcting for the reflection from a quartz window are given in column 2 of Table 3. In Fig. 2, curve B, the crosses (× ×) show the absorption in a plate of quartz 3.5 cm in thickness. They represent preliminary data, observed by Warburg [5] and his associates.

To obtain an accurate calibration curve of a prism is a laborious process and these data are included in Table 5 of the present paper. The calibration curve (minimum deviation settings of the spectrometer circle and wave lengths) was computed from the refractive indices of Carvallo [6] and of Paschen.[7] The fiducial line is the yellow helium line; $\lambda = 0.58758\mu$.

The data pertain to a quartz prism having a refracting angle of 59° 51′ 21″. Hence, for a prism having a considerably different angle the wave lengths will be found to be in error. However, the "slit-width factors" for reducing an energy curve from prismatic to normal spectrum will not be affected by the usual deviations from a 60° prism. These factors relate to a radiometer receiver 10′ in width, and they may be used for slit widths at least 50 per cent different from this value. For example, the test was made for a width of 6′, which should give slit-width

[5] Warburg, Leithäuser, Hupka, Müller, Ann. der Phys. (4), 40, p. 614; 1913.

[6] Carvallo, Compt. Rendus, 126, p. 728; 1898.

[7] Paschen, Ann. der Phys., (4), 35, p. 1005; 1911.

factors which are 0.6 of those given in Table 5. By reading the wave lengths from the calibration curve for a difference of 6′ on the spectrometer circle, the wave lengths subtended by a radiometer, 6′ in width, in different parts of the spectrum were found in agreement to within 2 parts in 10 000 with the values (i. e., 0.6 of the values) in Table 5, which is as accurate as one can read the values from the calibration curve.

TABLE 5

Quartz Prism Calibration

Spect. setting	Wave length	Slit width=10′	Spect. setting	Wave length	Slit width=10′	Spect. setting	Wave length	Slit width=10′
° ′	μ	μ	° ′	μ	μ	° ′	μ	μ
+2 30	0. 3206	......	15	0. 5269	0. 0340	−2 00	1. 9496	0. 1178
25	. 3242	0. 0075	10	. 5451	. 0385	05	2. 0073	. 1148
20	. 3281	. 0080	05	. 5654	. 0425	10	2. 0644	. 1118
15	. 3322	. 0082	0 00	. 58758	. 0465	15	2. 1191	. 1089
10	. 3363	. 0084	− 05	. 6119	. 0507	20	2. 1733	. 1061
05	. 3406	. 0089	10	. 6383	. 0562	25	2. 2252	. 1032
+2 00	. 3452	. 0093	15	. 6681	. 0637	30	2. 2765	. 1004
55	. 3500	. 0096	20	. 7020	. 0717	35	2. 3256	. 0978
50	. 3550	. 0100	25	. 7398	. 0808	40	2. 3743	. 0954
45	. 3600	. 0105	30	. 7828	. 0902	45	2. 4210	. 0932
40	. 3652	. 0109	35	. 8300	. 1004	50	2. 4675	. 0914
35	. 3708	. 0118	40	. 8832	. 1100	55	2. 5124	. 0896
30	. 3764	. 0122	45	. 9400	. 1193	−3 00	2. 5571	. 0878
25	. 3827	. 0128	50	1. 0025	. 1285	05	2. 6002	. 0862
20	. 3889	. 0135	55	1. 0685	. 1365	10	2. 6433	. 0845
15	. 3959	. 0144	−1 00	1. 1390	. 1418	15	2. 6847	. 0829
10	. 4029	. 0151	05	1. 2103	. 1445	20	2. 7262	. 0814
05	. 4108	. 0162	10	1. 2835	. 1454	25	2. 7661	. 0798
+1 00	. 4193	. 0176	15	1. 3557	. 1444	30	2. 8060	. 0784
55	. 4284	. 0185	20	1. 4279	. 1425	35	2. 8445	. 0771
50	. 4378	. 0195	25	1. 4982	. 1397	40	2. 8831	. 0758
45	. 4479	. 0213	30	1. 5676	. 1369	45	2. 9203	. 0746
40	. 4591	. 0226	35	1. 6351	. 1337	50	2. 9577	. 0735
35	. 4705	. 0239	40	1 7013	. 1304	55	2. 9938	. 0723
30	. 4830	. 0260	45	1. 7655	. 1274	−4 00	3. 0300	. 0712
25	. 4965	. 0281	50	1. 8287	. 1240	05	3. 0650	. 0703
20	. 5111	. 0304	55	1. 8895	. 1209	10	3. 1003	.....

The manner of obtaining the calibration curve and slit-width correction factors is given more fully elsewhere.[8] In Table 5 the "spectrometer setting" is the minimum deviation setting. If

[8] This Bulletin, 10, p. 2; 1913.

a Wadsworth mirror-prism device is used, the angular rotation is only half as great to attain the wave length here recorded.

In conclusion, acknowledgement is due to my assistant, W. B. Emerson, for the numerous computations involved in this investigation.

SUMMARY

This paper gives quantitative data on the absorption, reflection, and dispersion of quartz, extending from the ultra-violet to 3μ in the infra-red. The data may be used in determining spectral energy curves. Quartz is practically transparent from the ultra-violet to 1.8μ. It begins to absorb strongly beyond 1.8μ, and tabulated data are given for eliminating the effect of this absorption in a quartz prism. The results show that (within the errors of observation) in unpolarized light, the transmission is not affected by the direction in which the radiations pass through the material with respect to the optic axis.

WASHINGTON, December 9, 1913.